ATME College of Engineering

Department of EEE
Emitting Elite Energy

AY-2025-26
Batch: 2022-26

Power System Analysis-2
BEE703(IPCC)

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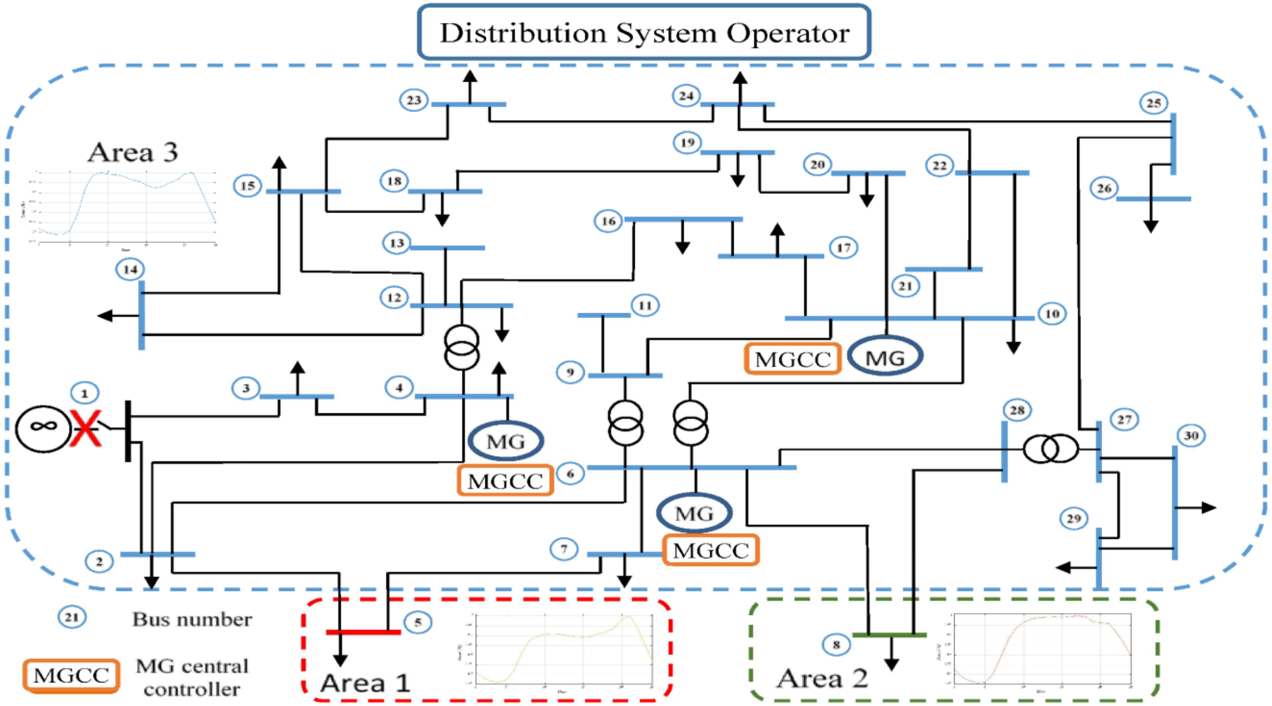
Module-1:Network Topology

1. Introduction
2. Basic definitions of Elementary graph theory
3. Tree, cut-set, loop.
4. Formation of Incidence Matrices.
5. Primitive network-Impedance form and admittance form,
6. Formation of Y Bus by Singular Transformation.
7. Y bus by Inspection Method.
8. Illustrative examples.

1.1 Introduction

- In Power System Analysis–2, we deal with the mathematical modeling and analysis of large electrical power networks
- We represent a complex power system as a network of buses and branches

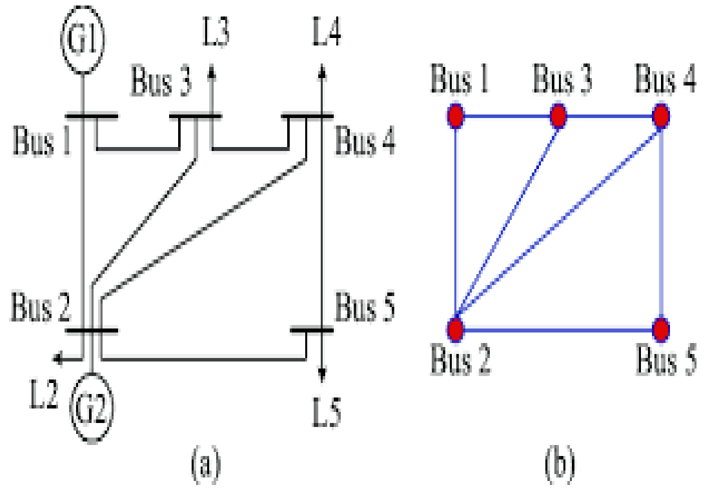
Analogy: Google Maps shows a city as a system of roads and junctions.



Power Network

1.1 Introduction cntd.

- Practically: Every generating station, transformer substation, and load center in the power grid acts as a bus (node)
- Transmission lines or cables - branches (links)
- To analyze such systems, we convert physical components like transformers, lines, and loads into mathematical models and use matrix methods like:



✓Bus Incidence Matrix (A) – to show how elements connect to buses
✓Bus Admittance Matrix (Y-bus) – to understand how currents and voltages flow in the network

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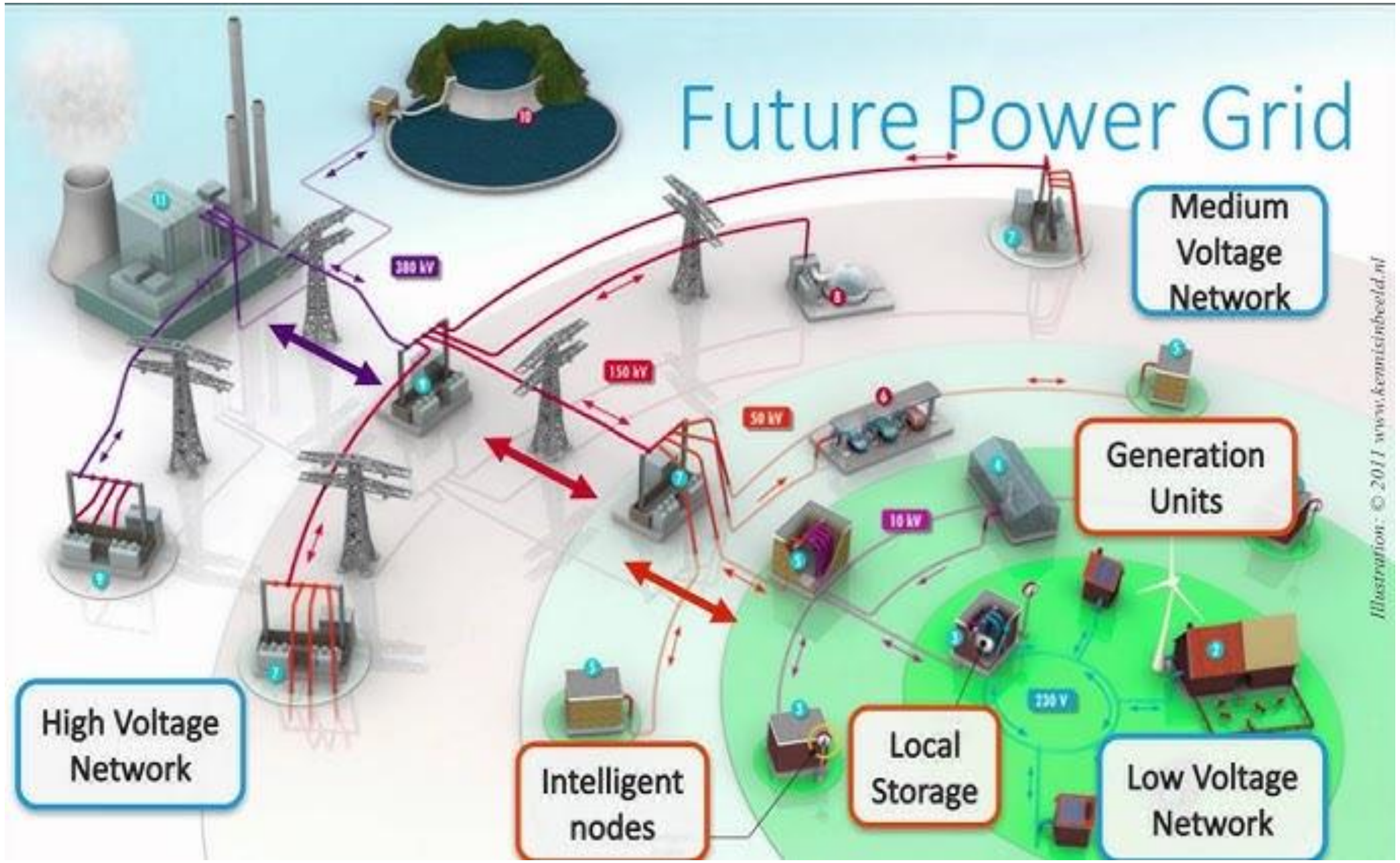
Why Power System Analysis is important?

This is essential in real life for:

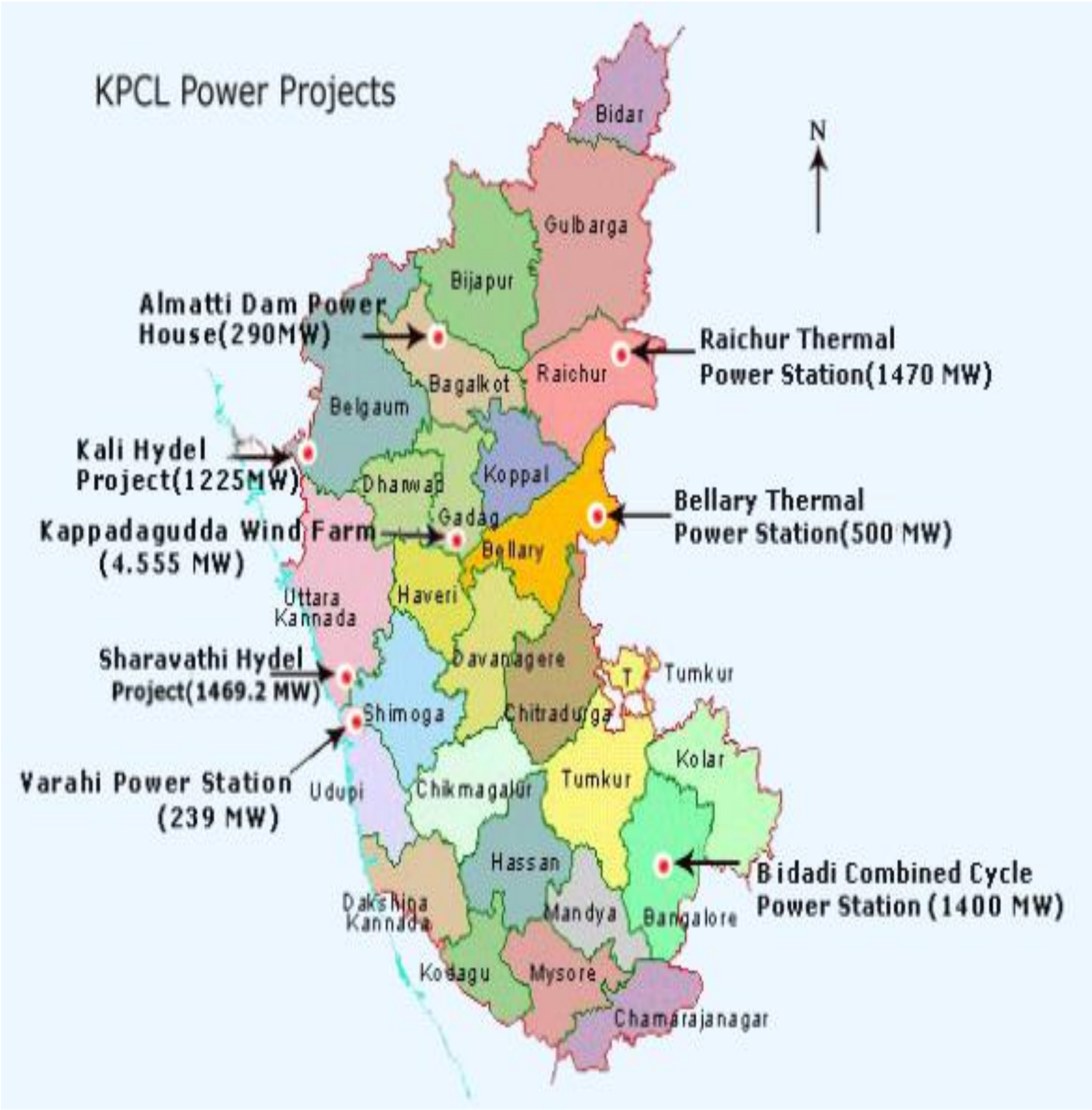
- Power flow studies (what happens when load increases)
- Fault analysis (what happens during short circuits)
- Stability checks (can the grid survive a disturbance?)



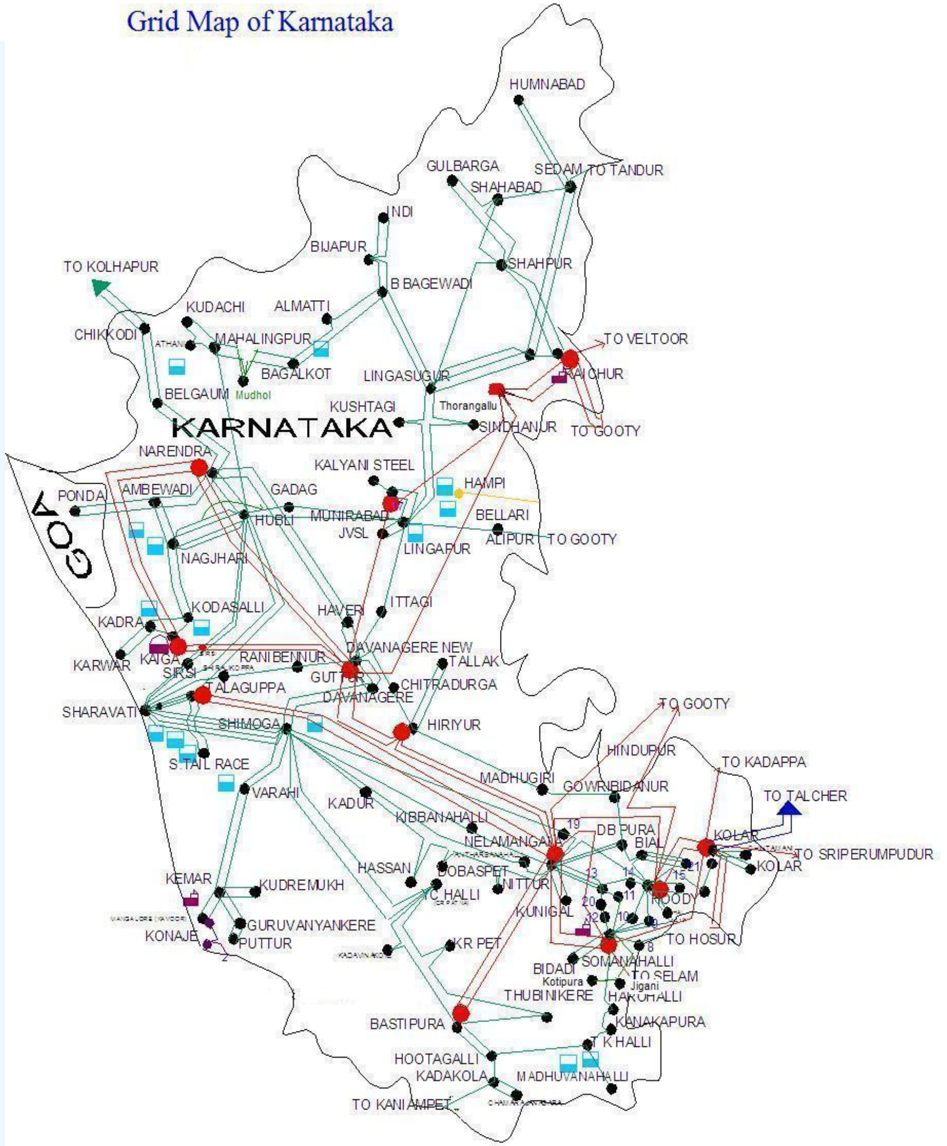
- Engineers in control centers monitor & control the power flow using these models.
- During real-time operation, software like SCADA or PSSE (Power System Simulator for Engineers)- uses Y-bus and power flow equations.
- Helps predict overloading and plan upgrades to prevent blackouts.

Real time example

- Imagine Karnataka's grid is receiving power from multiple sources (Sharavathi hydro, Raichur thermal, solar parks).
- Each one is a bus in your system.
- If one line fails, engineers use the Y-bus model to recalculate how power flows through alternate routes — instantly!

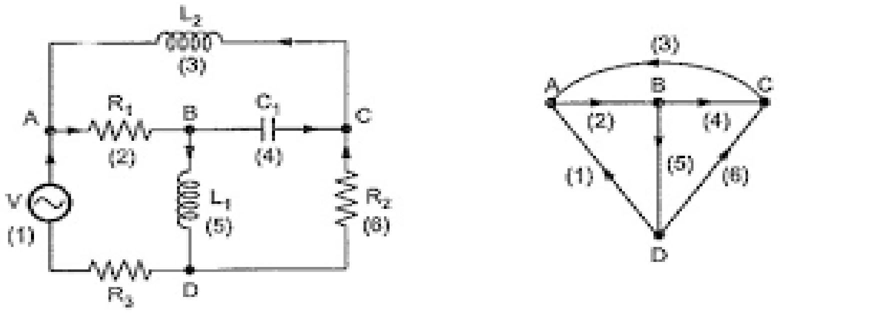


Grid Map of Karnataka



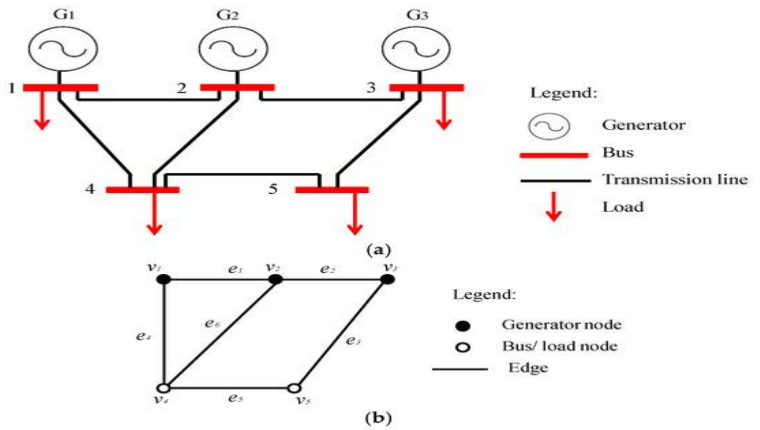
1.2 Elementary Graph Theory

- Graph: Geometrical interconnection of elements of a network

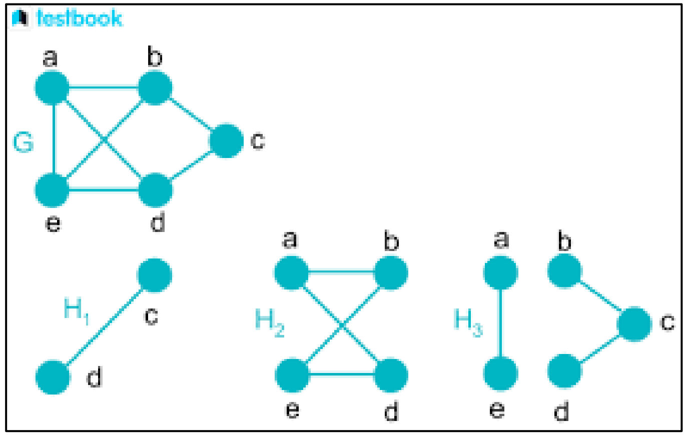


(a) Network (b) Oriented graph of a Network

Fig. 5.2 Network and oriented graph



Subgraph: Smaller part of larger subgraph

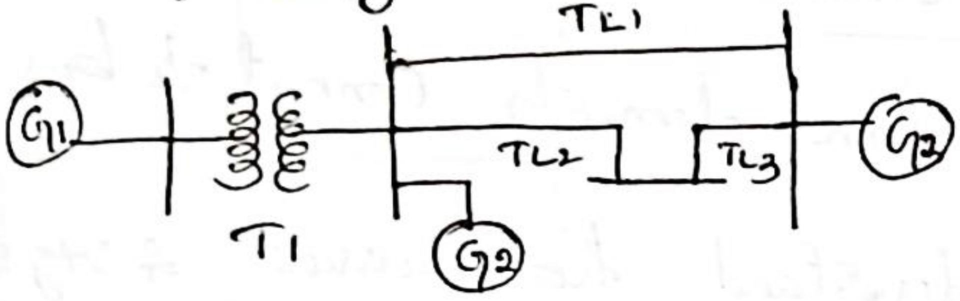


Path: It is a sequence of lines and nodes that connect two buses without repeating any node.

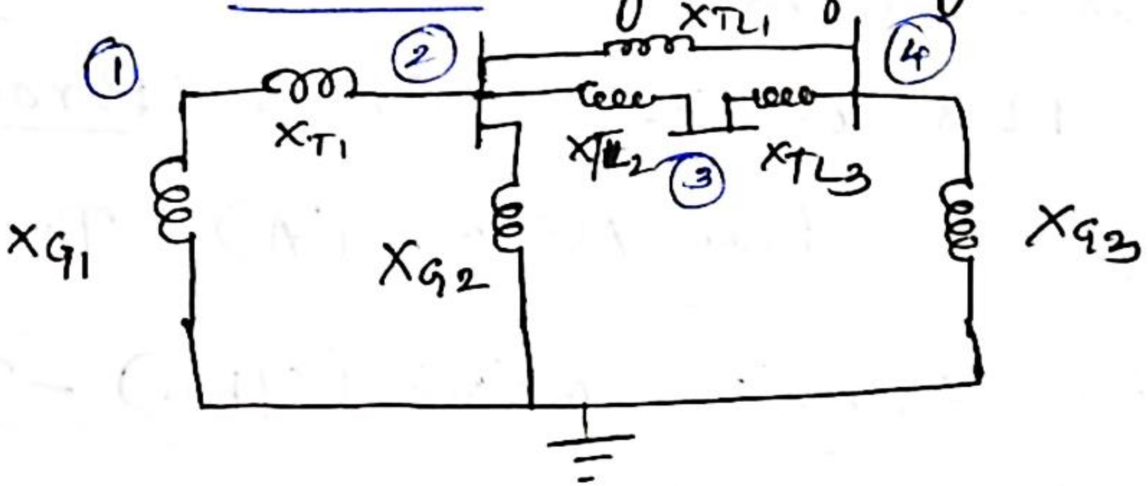
Oriented Graph: If each element of connected graph is assigned with a direction , then it is oriented graph.

1.3 Consider the following :

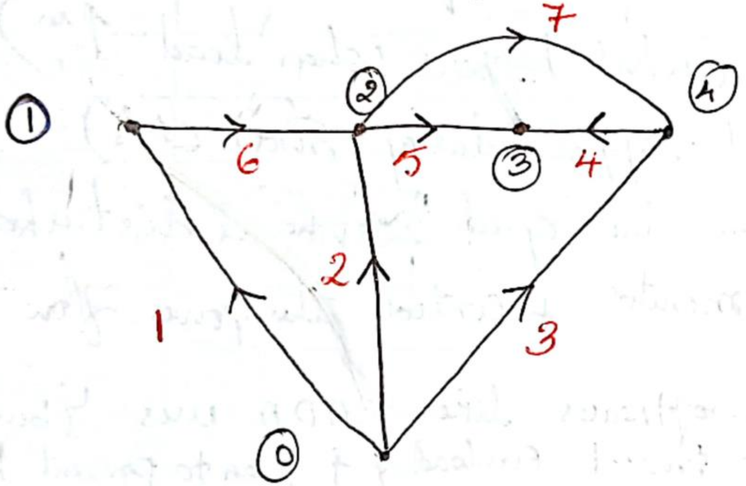
(1) Single line diagram



(2) Reactance diagram of fig 1



(3) Oriented graph



Let,

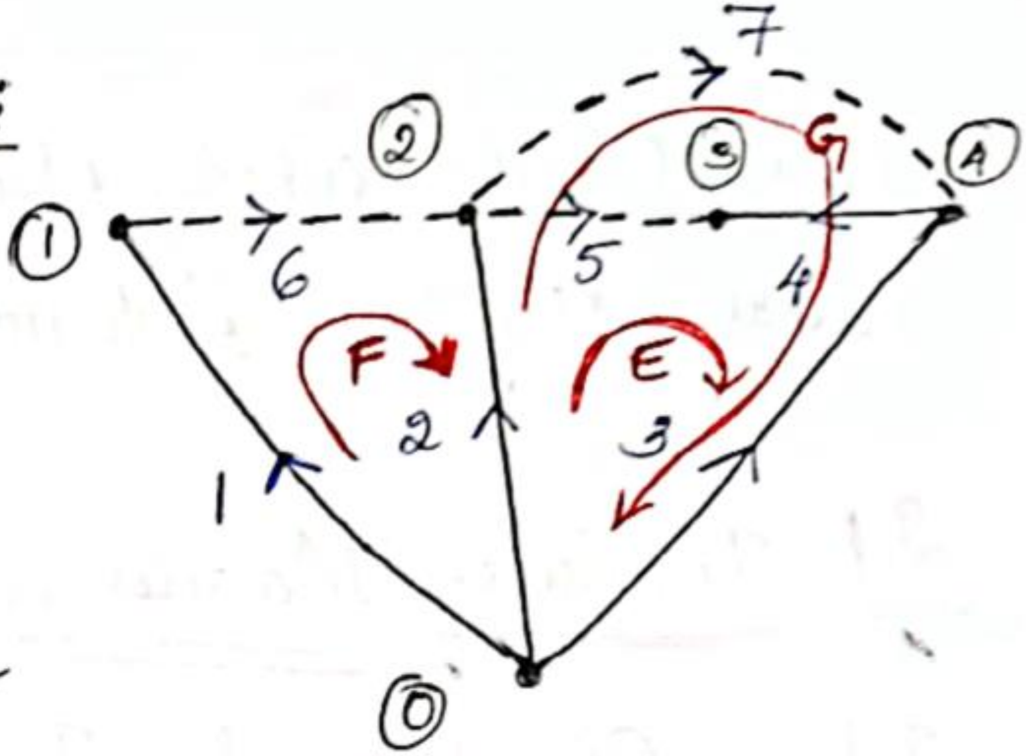
n - no. of nodes = 5

e - no. of lines or elements = 7

b - no. of branches

l - no. of links

1.4 Tree & Co-tree of the Oriented graph:



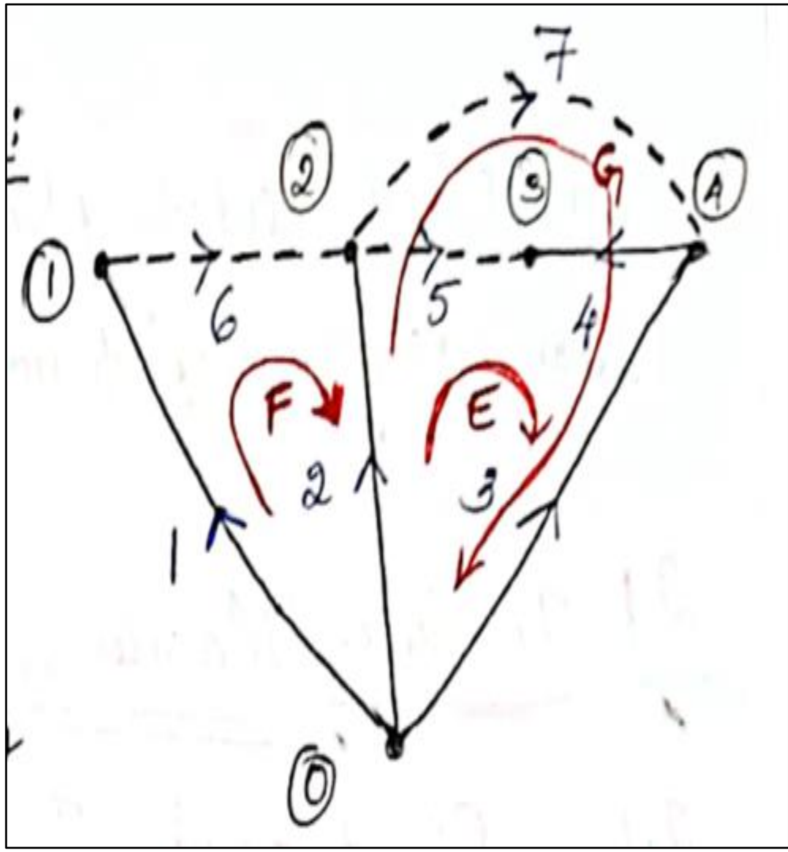
* Tree: A connected subgraph containing all nodes of a graph but no closed path.

* Branch: The elements of a tree are called branches & form a subset of the elements of the connected graph.

We have $\boxed{b = n-1}$; where n - no. of nodes = 5

$b = 5-1 = 4$ ⇒ Branches are 1, 2, 3, 4 (solid lines)

* Links: Those elements of the connected graph that are not included in the tree are called links & forms a subgraph
we have $\boxed{l = e - b}$ or $l = e-(n-1)$ or $\boxed{l = e-n+1}$
where e - no. of elements.
$\Rightarrow l = 7-4 = 3$
Links are 5, 6, 7 (dotted lines)



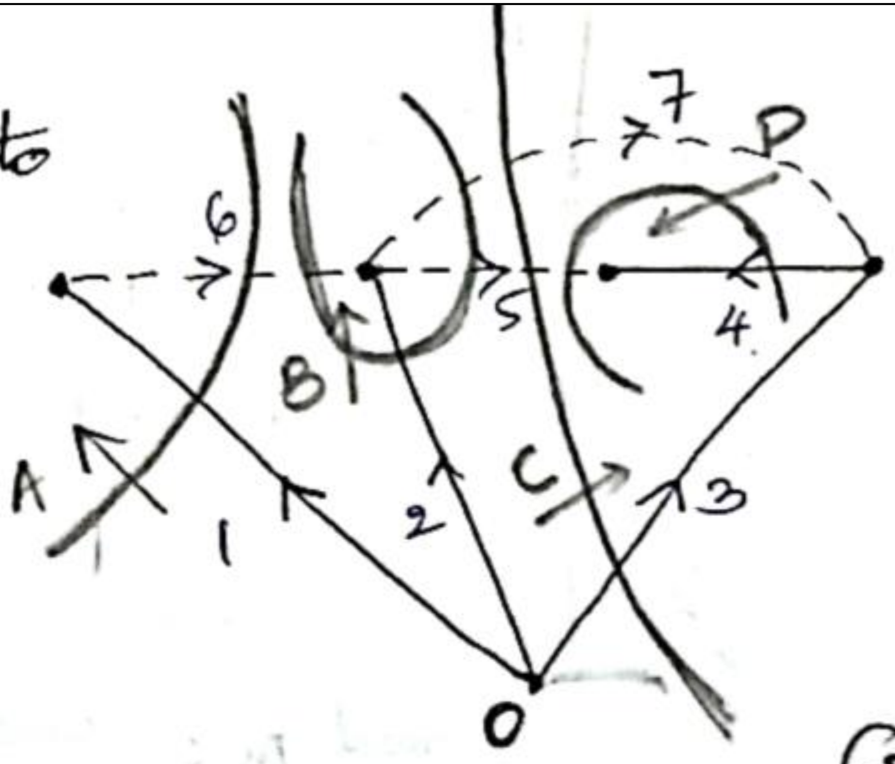
* Co-tree: It is compliment of tree. Links & co-tree are same.

* Loop: It is a closed path in a graph. It starts & ends at the same node. If a link is added to the tree, the resulting graph contains one closed path.

* Basic loop: The loops which contain only one link are independent & are called basic loops.
  * No. of loops = No. of links
  * Orientation of a basic loop is chosen - same as that of link

* **Cut-Set:** It is a set of elements that, if removed, divides a connected graph into two connected subgraphs

* **Twig:** Elements of a tree are called twigs or tree-branches



Direction of cut-set is same as that of branches

3

* **Basic Cut set:** Cut sets which contain only one twig & remaining links. They are equal to no. of twigs.

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2. Incidence Matrices:

2.1 Element- node Incidence Matrix or Branch node incidence matrix or bus incidence matrix ‘A’=(exn)

* The incidence of elements to nodes in a connected graph is shown by element-node incidence matrix

* The elements are obtained as follows.
  e – are placed on rows ; n – are placed on columns in matrix.

  $\boxed{a_{ij} = 1}$ → If i^{th} element is incident to & oriented away from node

  $\boxed{a_{ij} = -1}$ → If i^{th} element is incident to & oriented towards the node

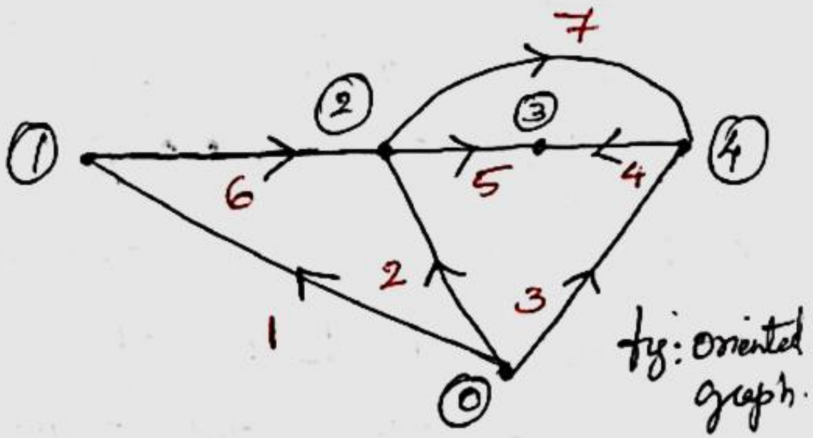
  $\boxed{a_{ij} = 0}$ → If i^{th} element is not incident to j^{th} node.

* Dimension of matrix is $\boxed{\hat{A} = e \times n}$

2.1 Element- node Incidence Matrix Cntd

* Dimension of matrix is $\boxed{\hat{A} = e \times n}$

* $\hat{A}$ for for oriented graph in 1.3 is



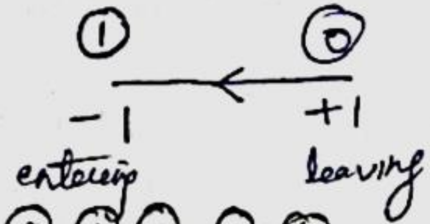
Nodes →

elements ↓

$\hat{A}$ =

e \ n	0	1	2	3	4
1	1	-1			
2	1		-1		
3	1				-1
4				-1	1
5			1	-1	
6		1	-1		
7			1		-1

Element node incidence matrix - $\hat{A}$



(a)

$$
\begin{array}{c|c}
 & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} &
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & -1 & 0 \\
0 & 1 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & -1
\end{bmatrix}
\end{array}
$$

(4)

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2.1 Bus Incidence Matrix

* Any nodes of connected graph can be chosen as reference node & the column corresponding to that node is deleted from $\hat{A}$. The matrix so formed is called the Bus-incidence matrix A.

* Dimension of $A = e \times (n-1)$; where $(n-1)$ – rank of matrix.

* From above example consider the node ⓪ as reference node. We get matrix A given by,

$$A = \begin{array}{c|c} e \backslash n & \begin{matrix} ① & ② & ③ & ④ \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ \hline 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array} = \begin{array}{c|c} e \backslash n & \\ \begin{matrix} \text{Branches} \\ \text{Links} \end{matrix} & \begin{bmatrix} A_b \\ \hline A_l \end{bmatrix} \end{array}$$

* Above matrix is partitioned into 2 submatrices as shown above:

$A_b = b \times (n-1)$

$A_l = l \times (n-1)$

2.2 Branch- path Incidence matrix 'k'=(bxpaths)

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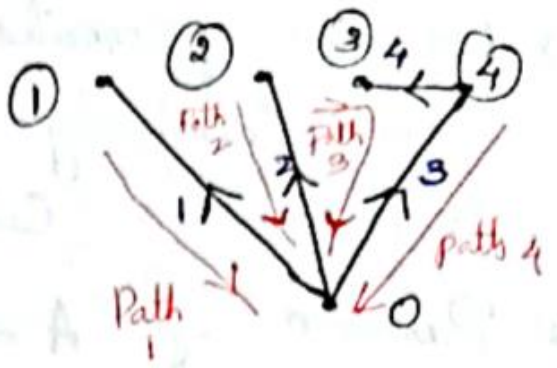
* The incidence of branches to paths in a tree is shown by the branch-path incidence matrix, where a path is oriented from bus to reference node.

* Elements of the matrix are:
  - $\boxed{K_{ij}=1}$ → If the i^{th} branch is in the path from j^{th} bus to reference & is oriented in same direction.
  - $\boxed{K_{ij}=-1}$ → If the i^{th} branch is in the path from j^{th} bus to reference but oriented in opposite direction.
  - $K_{ij}=0$ → If the i^{th} branch is not in the path from j^{th} bus to reference.

* $n=5$, $b=n-1=5-1=4$. Where b – branch.
* $K = b \times path$.

2.2 Branch- path Incidence matrix 'k' cntd.

$$K = \begin{array}{c|cccc} b \backslash \text{Path} & 1 & 2 & 3 & 4 \\ \hline 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & -1 \\ 4 & 0 & 0 & -1 & 0 \end{array}$$



* Branch-path incidence matrix & Submatrix A_b relates the branches to paths & branches to buses resp.

$$\therefore A_b \cdot K^T = U$$

$$\therefore \boxed{K^T = [A_b]^{-1}}$$

2.3 Basic Cut-set Incidence matrix 'B'=(exbc) cntd.

* The incidence of elements to basic cut sets of a connected graph is shown by the basic cut-set incidence matrix 'B':

* The elements of the matrix are:
  - $\boxed{b_{ij} = 1}$ → If i^{th} element is incident to & oriented in the same direction as the j^{th} basic cut-set
  - $\boxed{b_{ij} = -1}$ → If i^{th} element is incident to & oriented in opposite direction as the j^{th} basic cut set
  - $\boxed{b_{ij} = 0}$ → If i^{th} element is not incident to j^{th} basic cut set

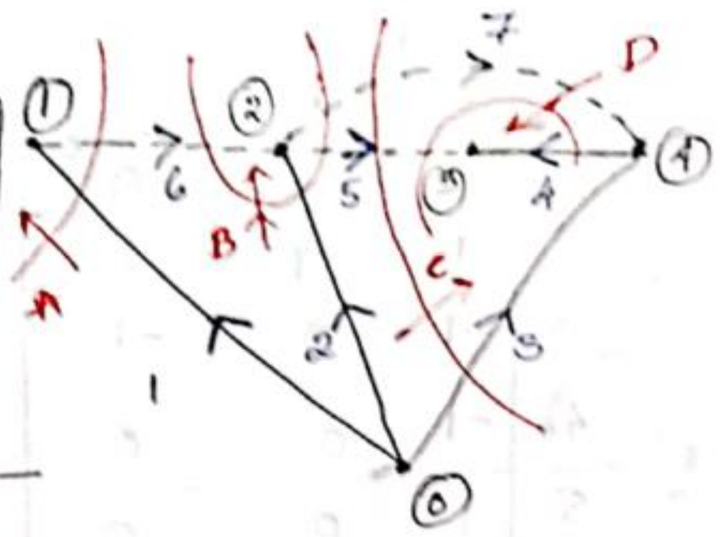
* Basic cut-set incidence matrix dimension is $B = e \times b$.

* Consider the cut-set graph shown previously.

2.3 Basic Cut-set Incidence matrix ‘B=(e x bc)’ cntd.

Basic cut-sets

$$
B = \begin{array}{c|cccc}
e \backslash b & A & B & C & D \\
\hline
1 & 1 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 \\
3 & 0 & 0 & 1 & 0 \\
4 & 0 & 0 & 0 & 1 \\
\hline
5 & 0 & -1 & 1 & 1 \\
6 & -1 & 1 & 0 & 0 \\
7 & 0 & -1 & 1 & 0
\end{array}
= \begin{array}{c}
\text{Branches} \\
\\
\text{Links}
\end{array}
\begin{bmatrix} U_B \\ \hline B_l \end{bmatrix}
$$



Here $U_B \rightarrow$ Identity Matrix $\rightarrow$ Corresponds to Branches of cut-sets.

* $B_l \rightarrow$ Can be obtained from Bus incidence Matrix as follows.

$$B_l \cdot A_b = A_l$$

$$B_l = A_l \cdot A_b^{-1}$$

WKT $\therefore A_b^{-1} = K^T$

$\therefore \boxed{B_l = A_l \cdot K^T} \rightarrow B_l$ in terms of Bus incidence matrix.

2.3 Basic Loop Incidence matrix ‘C’=(exbl) cntd.

* The incidence of elements to basic loops of connected graph is shown by basic loop incidence matrix C

* The elements of the matrix are:

  $C_{ij} = 1$ → If the i^{th} element is incident to & oriented in the same direction as j^{th} basic loop.

  $C_{ij} = -1$ → If the i^{th} element is incident to & oriented in opposite direction as j^{th} basic loop.

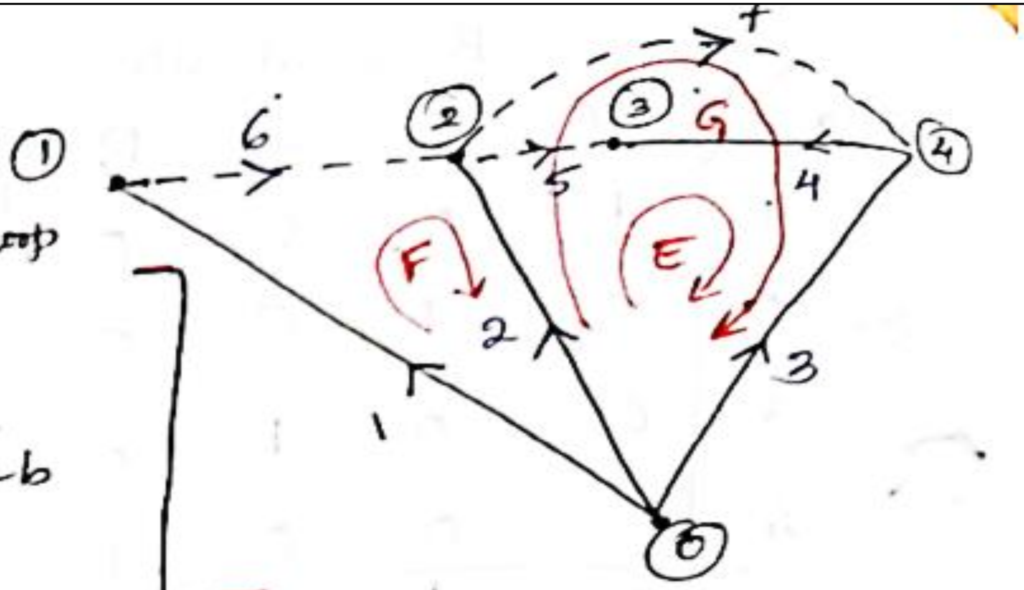
  $C_{ij} = 0$ → If the i^{th} element is not incident to j^{th} basic loop.

* The basic loop incidence matrix dimension is $C = e \times l$.

2.3 Basic Loop Incidence matrix 'C'=(e x bl) cntd.

Basic loops

$$
C = \begin{array}{c|c}
 e \backslash l & \begin{array}{ccc} E & F & G \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} &
\left[\begin{array}{ccc}
0 & 1 & 0 \\
1 & -1 & 1 \\
-1 & 0 & -1 \\
-1 & 0 & 0 \\ \hline
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]
\end{array}
=
\begin{array}{c|c}
 e \backslash l & \text{Basic loop} \\
\text{Branches} & \left[\begin{array}{c} C_b \\ \hline U_l \end{array}\right] \\
\text{Links} &
\end{array}
$$



where U_l – Identity Matrix